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Stochastic resonance in linear system due to dichotomous noise modulated by bias signal

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Abstract

The stochastic resonance in an over-damped linear system due to dichotomous noise modulated by a bias signal is studied in detail. By the theory of signal-to-noise ratio (SNR) and the Shapiro–Loginov formula, the exact expressions of the first two moments and SNR for the output to the asymmetric dichotomous noise input are obtained. It is found that each curve of the SNR versus the multiplicative noise intensity exhibits a mono peak and the conventional stochastic resonance appears, which is absent for the case of noise and periodic signal introduced additively. Meanwhile, the SNR is a non-monotonic function of the signal frequency or the correlation time of noise, and the bona fide stochastic resonance (SR) and SR in a broad sense exist. Moreover, the SNR depends on the additive noise intensity, cross-correlation strength and asymmetry of multiplicative noise.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Benzi *et al* [1] and Nicolis *et al* [2] first proposed 'stochastic resonance' (SR) to explain the periodic switching of the earth's climate between ice ages and warm ages. SR is a phenomenon in which the response of a dynamical system to an input signal is enhanced by the addition of an optimal amount of noise. This counterintuitive phenomenon attracted lots of people and has been extensively investigated both theoretically and experimentally [3–8]. Fauve *et al* [3] and McNamara *et al* [4] observed SR in an experiment of the Schmitt trigger and the bistable ring laser. McNamara *et al* [4, 5] suggested a two-state model and obtained the SNR under

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adiabatic limit to characterize SR. Dykman *et al* [6] and Hu *et al* [7] introduced the linear-response theory and perturbation theory to investigate SR. Zhou and Moss [8] employed the residence-time distribution to explain SR as a resonance synchronization phenomenon. But most of these studies considered the nonlinearity to be an essential ingredient for the presence of SR. However, in recent years SR was also found in linear systems driven by multiplicative coloured noise or dichotomous noise [9–13]. Berdichevsky *et al* [11] pointed out that the SNR is a non-monotonic function of some characteristics of the noise, but not of the noise intensity. So we care about whether or not the conventional SR appearing in the linear system, that is, the SNR, is a non-monotonic function of noise intensity.

Generally, the external noise and the weak periodic force are introduced additively in most of the above-mentioned papers. However, noise modulated by periodical signal does occur at the output of amplifiers in optics or radio astronomy and has an important effect. So Dykman *et al* [14] studied an asymmetric bistable system when the noise and periodic signal were introduced multiplicatively, namely, signal-modulated noise. They detected the existence of SR in such an asymmetric bistable system and verified their results by an electronic analog experiment. In practice there are two kinds of signal-modulated noise: one is the direct signal modulation; the other is the bias signal modulation. The bias signal modulation is widely applied to the modern communication system [15] and improves the quality of communication. And the modulated process is operated in the linear region for avoiding distortion of a modulation signal. Yet only few theoretical papers involve the effect of the bias signal-modulated noise on the stochastic linear system.

In this paper, we investigate the stochastic resonance of an over-damped linear system with bias signal-modulated noise and find some novel nonlinear phenomena. The paper is organized as follows. In section 2, we obtain the exact expressions of the first two moments and the signal-to-noise ratio of the linear system for the case of asymmetric dichotomous noise. In section 3, by analysing the numerical results we provide a discussion and draw some conclusions. It should be pointed out that the SR has been introduced in a broad sense in [11]; so here we only need to investigate the non-monotonic behaviour of the output signal as a function of noise intensity and the other characteristics of the noise. Bona fide SR means that the curve of SNR exhibits a resonance peak with increasing frequency of signal. Moreover, the conventional SR means that the curve of SNR exhibits a single peak with the increase of noise intensity. Here the expression of SNR can be applied to arbitrary noise intensity and signal amplitude without being restricted to the condition of adiabatic limit.

2. The output signal-to-noise ratio of a linear system with bias signal-modulated dichotomous noise

Consider an over-damped linear system with an input dichotomous noise modulated by a bias signal, the output of which is described by the following equation:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -(a+\xi(t))x + \left(\alpha + \frac{1}{2}A_0\cos\Omega t\right)\eta(t), \tag{1}$$

where A_0 is the amplitude of a periodic signal, Ω is the frequency, α can only take either of the two values 0 and 1; $\alpha = 0$ denotes the direct signal-modulated noise and $\alpha = 1$ denotes the bias signal-modulated noise. $\xi(t)$ and $\eta(t)$ are dichotomous noise [19], namely a kind of asymmetric, two-state random process, with zero mean and correlation functions described as follows:

$$\begin{aligned} &\langle\xi(t)\rangle = \langle\eta(t)\rangle = 0, \qquad \langle\xi(t)\xi(t')\rangle = \sigma_1\exp(-\lambda|t-t'|), \\ &\langle\eta(t)\eta(t')\rangle = \sigma_2\exp(-\lambda|t-t'|). \end{aligned} \tag{2}$$

Here $\xi(t)$ and $\eta(t)$ only have two values respectively. For example

$$\begin{aligned} &\xi(t) \in \{M_1, -N_1\}, \qquad \sigma_1 = M_1 N_1, \qquad \lambda = p_1 + p_2, \qquad \Delta_1 = M_1 - N_1, \\ &\eta(t) \in \{M_2, -N_2\}, \qquad \sigma_2 = M_2 N_2, \qquad \lambda = q_1 + q_2, \qquad \Delta_2 = M_2 - N_2, \end{aligned} \tag{3}$$

where p_1 is the transition rate of $\xi(t)$ from M_1 to $-N_1$ and p_2 is the reverse rate. q_1 is the transition rate of $\eta(t)$ from M_2 to $-N_2$ and q_2 is the reverse one. Δ_1 and Δ_2 denote the asymmetry of the dichotomous noise $\xi(t)$ and $\eta(t)$ respectively.

Now we assume that $\xi(t)$ and $\eta(t)$ are actually of the same random source; then some form of correlation exists between them, i.e.

$$\langle \xi(t)\eta(t')\rangle = \langle \eta(t)\xi(t')\rangle = \sigma_3 \exp(-\lambda|t-t'|), \tag{4}$$

where $\sigma_3 = \sigma_1 r + \sigma_2 q$, r and q are confined to the 0–1 interval [16], measuring the contribution of each individual noise respectively.

Taking the average on equation (1), and after multiplying equation (1) by $2x$ then taking the average, we obtain the first two moments in the following form:

$$\frac{\mathrm{d}\langle x\rangle}{\mathrm{d}t} = -a\langle x\rangle - \langle \xi(t)x\rangle, \tag{5}$$

$$\frac{\mathrm{d}\langle x^2\rangle}{\mathrm{d}t} = -2a\langle x^2\rangle - 2\langle \xi(t)x^2\rangle + (2\alpha + A_0\cos(\Omega t))\langle x\eta(t)\rangle. \tag{6}$$

Using the Shapiro–Loginov formula [17], we obtain the following equation:

$$\frac{\mathrm{d}\langle \xi x\rangle}{\mathrm{d}t} = \left\langle \xi \frac{\mathrm{d}x}{\mathrm{d}t}\right\rangle - \lambda\langle \xi x\rangle. \tag{7}$$

Multiplying equation (1) by $\xi(t)$ and combining it with equation (7), we obtain

$$\frac{\mathrm{d}\langle \xi x\rangle}{\mathrm{d}t} = -(a+\lambda)\langle \xi x\rangle - \langle \xi^2 x\rangle + \left(\alpha + \frac{1}{2}A_0\cos(\Omega t)\right)\sigma_3. \tag{8}$$

Since equation (8) contains the moment $\langle \xi^2 x\rangle$, we use the properties of dichotomous noise $\xi(t)$ to decouple this term:

$$\xi^2(t) = \sigma_1 + \Delta_1\xi(t). \tag{9}$$

According to equation (9), the lower-order moment $\langle \xi(t)x\rangle$ can express the higher-order one $\langle \xi^2 x\rangle$ as follows:

$$\langle \xi^2 x\rangle = \sigma_1\langle x\rangle + \Delta_1\langle \xi x\rangle. \tag{10}$$

Substituting equation (10) into equation (8), one obtains the following equation:

$$\frac{\mathrm{d}\langle \xi x\rangle}{\mathrm{d}t} = -(a+\lambda+\Delta_1)\langle \xi x\rangle - \sigma_1\langle x\rangle + \left(\alpha + \frac{1}{2}A_0\cos(\Omega t)\right)\sigma_3. \tag{11}$$

By virtue of equations (5) and (11), we obtain the differential equations of unknown functions $\langle x\rangle$ and $\langle \xi x\rangle$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\langle x\rangle \\ \langle \xi x\rangle\end{bmatrix} = \begin{bmatrix}-a & -1 \\ -\sigma_1 & -(a+\lambda+\Delta_1)\end{bmatrix}\begin{bmatrix}\langle x\rangle \\ \langle \xi x\rangle\end{bmatrix} + \begin{bmatrix}0 \\ \left(\alpha + \frac{1}{2}A_0\cos\Omega t\right)\sigma_3\end{bmatrix}. \tag{12}$$

By solving equation (12), one obtains the asymptotic value at $t \to \infty$ of the first moment $\langle x\rangle$,

$$\langle x\rangle = A_0\sigma_3\frac{f_1\cos(\Omega t) + f_2\sin(\Omega t)}{2f_3} - \alpha f_4, \tag{13}$$

where

$$f_1 = \Omega^2 - r_1 r_2, \qquad f_2 = -\Omega(r_1 + r_2), \qquad f_3 = \left(\Omega^2 + r_2^2\right)\left(\Omega^2 + r_1^2\right), \qquad f_4 = \frac{\sigma_3}{r_1 r_2},$$

$$r_{1,2} = a + \varepsilon_{1,2} = a + \frac{\lambda + \Delta_1}{2} \pm \sqrt{\frac{(\lambda + \Delta_1)^2}{4} + \sigma_1}.$$

Using the similar method and the Shapiro–Loginov formula, we obtain the following differential equations to get the second moment $\langle x^2 \rangle$:

$$\frac{\mathrm{d}\langle \xi x^2 \rangle}{\mathrm{d}t} = -(2a + \lambda + 2\Delta_1)\langle \xi x^2 \rangle - 2\sigma_1 \langle x^2 \rangle + (2\alpha + A_0 \cos(\Omega t))\langle \xi \eta x \rangle, \tag{14}$$

$$\frac{\mathrm{d}\langle x\eta \rangle}{\mathrm{d}t} = -(a + \lambda)\langle x\eta \rangle - \langle \xi \eta x \rangle + \left(\alpha + \frac{1}{2}A_0 \cos(\Omega t)\right)\sigma_2, \tag{15}$$

$$\frac{\mathrm{d}\langle \xi \eta x \rangle}{\mathrm{d}t} = -(a + \Delta_1 + 2\lambda)\langle \xi \eta x \rangle - \sigma_1 \langle x\eta \rangle + \left(\alpha + \frac{1}{2}A_0 \cos(\Omega t)\right)\sigma_3 \Delta_2. \tag{16}$$

According to equations (6) and (14)–(16), one obtains four differential equations of the unknown functions $\langle x^2 \rangle$, $\langle \xi x^2 \rangle$, $\langle x\eta \rangle$ and $\langle \xi \eta x \rangle$. Since the solution of these equations is rather complicated, here we give only the asymptotic value ($t \to \infty$) of the second moment:

$$\begin{aligned} \langle x^2 \rangle_{\mathrm{st}} = \{&\alpha[(a + \lambda/2 + \Delta_1)(\sigma_2 f_5 + \Delta_2 \sigma_3 f_6) - (\varepsilon_2 \sigma_2 f_5 + \varepsilon_1 \Delta_2 \sigma_3 f_6)] \\ &- (\varepsilon_2 \sigma_2 f_5 + \varepsilon_1 \Delta_2 \sigma_3 f_6) A_0/2\} \\ &\times \left\{2\left(r_3^2 + \Omega^2\right)\left(r_4^2 + \Omega^2\right)[a(2a + \lambda + 2\Delta_1) - 2\sigma_1]\right\}^{-1}, \end{aligned} \tag{17}$$

where

$$r_{3,4} = 2a + \frac{\lambda + 2\Delta_1}{2} \pm \sqrt{\frac{(\lambda + 2\Delta_1)^2}{4} + 4\sigma_1}, \qquad f_5 = \frac{\alpha}{r_2 + \lambda} + \frac{A_0 \Omega}{2[(r_2 + \lambda)^2 + \Omega^2]},$$

$$f_6 = \frac{\alpha}{r_1 + \lambda} + \frac{A_0 \Omega}{2[(r_1 + \lambda)^2 + \Omega^2]},$$

and ε_i, $r_i (i = 1, 2)$ were defined earlier.

Our aim is to obtain the stationary correlation function $\langle x(t + \tau)x(t) \rangle$. Therefore, from the linear equation (1) we can present the following formula:

$$\begin{aligned} x(t + \tau) = x(t)g(\tau)\exp(-a\tau) &+ \frac{A_0}{2}\int_0^\tau \exp(-av)h(v)\cos[\Omega(t + \tau - v)]\,\mathrm{d}v \\ &+ \alpha \int_0^\tau \exp(-av)h(v)\,\mathrm{d}v, \end{aligned} \tag{18}$$

where

$$g(v) = \left\langle \exp\left[-\int_0^v \xi(u)\,\mathrm{d}u\right]\right\rangle, \qquad h(t - v) = \left\langle \eta(v)\exp\left[-\int_v^t \xi(u)\,\mathrm{d}u\right]\right\rangle.$$

Equation (18) contains several integrals that involve the average value of the exponential of an integral of multiplicative noise. According to the defined master equation of the non-Markovian dichotomous noise and equations (2)–(4), the following approximation [18–20] has been used to derive the correlation function $\langle x(t + \tau)x(t) \rangle$. Here the approximation is valid for arbitrary $t > v$,

$$g(v) = \frac{\varepsilon_1}{\varepsilon_1 - \varepsilon_2}\exp[-\varepsilon_2 v] - \frac{\varepsilon_2}{\varepsilon_1 - \varepsilon_2}\exp[-\varepsilon_1 v], \tag{19}$$

$$h(t - v) = \frac{\sigma_3}{\lambda}\exp[-\lambda(t - v) - 1]g(t - v). \tag{20}$$

Multiplying equation (18) by $x(t)$ and performing average, we obtain the expression of the correlation function

$$\langle x(t+\tau)x(t)\rangle = \langle x^2\rangle_{\mathrm{st}}\, g(\tau)\exp(-a\tau) + \frac{\langle x\rangle A_0\sigma_3 \exp(-at-1)}{2\lambda(\varepsilon_1-\varepsilon_2)} \times [f_7\cos(\Omega t) + f_8\sin(\Omega t)] + \alpha\langle x\rangle \int_0^\tau \exp(-av)h(v)\,\mathrm{d}v, \tag{21}$$

where

$$f_7 = \exp[-(\lambda-r_2)t]\frac{-\varepsilon_1(\lambda-r_2)f_{12}+\varepsilon_1\Omega\sin(\Omega\tau)}{(\lambda-r_2)^2+\Omega^2} + \exp[-(\lambda-r_1)t]\frac{\varepsilon_2(\lambda-r_1)f_{10}-\varepsilon_2\Omega\sin(\Omega\tau)}{(\lambda-r_2)^2+\Omega^2}, \tag{22}$$

$$f_8 = \exp[-(\lambda-r_2)t]\frac{\varepsilon_1(\lambda-r_2)\sin(\Omega\tau)+\varepsilon_1\Omega f_{11}}{(\lambda-r_2)^2+\Omega^2} + \exp[-(\lambda-r_1)t]\frac{-\varepsilon_2(\lambda-r_1)\sin(\Omega\tau)-\varepsilon_2\Omega f_9}{(\lambda-r_2)^2+\Omega^2} \tag{23}$$

and

$$f_{9,10} = \cos(\Omega\tau) \pm \exp[(\lambda-r_1)\tau], \qquad f_{11,12} = \cos(\Omega\tau) \pm \exp[(\lambda-r_2)\tau]. \tag{24}$$

According to expression (21), the asymptotic value of the correlation function is obtained by averaging equation (21) over the period $2\pi/\Omega$:

$$\begin{aligned}\langle x(t+\tau)x(t)\rangle_{\mathrm{st}} &= \frac{\Omega}{2\pi}\int_0^{\frac{2\pi}{\Omega}} \langle x(t)x(t+\tau)\rangle\,\mathrm{d}t = \langle x^2\rangle_{\mathrm{st}}\, g(\tau)\exp(-a\tau) + \frac{A_0^2\sigma_3^2}{4\lambda(\varepsilon_1-\varepsilon_2)f_3}\\ &\times\{\varphi_5[f_1\varphi_1(\lambda+\varepsilon_2)^2 - \Omega(\lambda+\varepsilon_2)(f_1\varphi_3+f_2\varphi_1) + 2\Omega^2(f_1\varphi_1 - f_2\varphi_3)]\\ &+\varphi_6[f_1\varphi_2(\lambda+\varepsilon_1)^2 - \Omega(\lambda+\varepsilon_1)(f_1\varphi_4+f_2\varphi_2) + 2\Omega^2(f_1\varphi_2 - f_2\varphi_4)]\}\\ &-\alpha f_4\sigma_3/\lambda(\varepsilon_1-\varepsilon_2)\left\{\frac{r_1\,\mathrm{e}^{-1}}{a+\lambda+r_2}[1-\exp\{-(a+\lambda+r_2)\tau\}]\right.\\ &\left.-\frac{r_2\,\mathrm{e}^{-1}}{a+\lambda+r_1}[1-\exp\{-(a+\lambda+r_1)\tau\}]\right\},\end{aligned} \tag{25}$$

where

$$\varphi_1 = \frac{-\varepsilon_1(\lambda-r_2)f_{12}+\varepsilon_1\Omega\sin(\Omega\tau)}{(\lambda-r_2)^2+\Omega^2}, \qquad \varphi_2 = \frac{\varepsilon_2(\lambda-r_1)f_{10}-\varepsilon_2\Omega\sin(\Omega\tau)}{(\lambda-r_2)^2+\Omega^2},$$

$$\varphi_3 = \frac{\varepsilon_1(\lambda-r_2)\sin(\Omega\tau)+\varepsilon_1\Omega f_{11}}{(\lambda-r_2)^2+\Omega^2}, \qquad \varphi_4 = \frac{-\varepsilon_2(\lambda-r_1)\sin(\Omega\tau)-\varepsilon_2\Omega f_9}{(\lambda-r_2)^2+\Omega^2},$$

$$\varphi_5 = \frac{1}{(\lambda+\varepsilon_2)[(\lambda+\varepsilon_2)^2+4\Omega^2]}, \qquad \varphi_6 = \frac{1}{(\lambda+\varepsilon_1)[(\lambda+\varepsilon_1)^2+4\Omega^2]}.$$

We perform the Fourier transform on equation (25) and obtain the power spectrum $S(\omega)$ for positive ω as follows:

$$\begin{aligned}S(\omega) &= \int_{-\infty}^{\infty}\langle x(t)x(t+\tau)\rangle_{\mathrm{st}}\exp(-\mathrm{i}\omega\tau)\,\mathrm{d}\tau\\ &= S_0\delta(\omega) + S_1(\omega) + S_2(\omega)\delta(\omega-\Omega) + (r_1 \leftrightarrow r_2),\end{aligned} \tag{26}$$

where

$$S_0 = \frac{2\,\mathrm{e}^{-1}\pi\alpha f_4\sigma_3}{\lambda(\varepsilon_1-\varepsilon_2)^2}\left[\frac{r_2}{a+\lambda+r_1}-\frac{r_1}{a+\lambda+r_2}\right],$$

$$\begin{aligned} S_1(\omega) = {}& \frac{\langle x^2\rangle_{\mathrm{st}}}{\varepsilon_1-\varepsilon_2}\left[\frac{\varepsilon_2 r_1}{r_1^2+\omega^2}-\frac{\varepsilon_1 r_2}{r_2^2+\omega^2}\right] \\ & - \frac{A_0^2\sigma_3^2[\varphi_5\varepsilon_1(f_1A_1+f_2\Omega A_2)+\varphi_6\varepsilon_2(f_1A_3+f_2\Omega A_4)]}{4\lambda f_3(\varepsilon_1-\varepsilon_2)[(\lambda-r_2)^2+\Omega^2]} \\ & + \frac{2\,\mathrm{e}^{-1}\alpha f_4\sigma_3}{\lambda(\varepsilon_1-\varepsilon_2)}\left[\frac{a+\lambda+r_2}{(a+\lambda+r_2)^2+\omega^2}-\frac{a+\lambda+r_1}{(a+\lambda+r_1)^2+\omega^2}\right], \end{aligned} \tag{27}$$

$$S_2(\omega) = \frac{\pi A_0^2\sigma_3^2[\varphi_5\varepsilon_1(f_1B_1+f_2\Omega B_2)+\varphi_6\varepsilon_2(f_1B_3+f_2\Omega B_4)]}{4\lambda f_3(\varepsilon_1-\varepsilon_2)[(\lambda-r_2)^2+\Omega^2]}. \tag{28}$$

The symbols in equations (26)–(28) are given by the following expressions:

$$\begin{aligned} A_1 &= \Omega^2(\varepsilon_2+2r_2-\lambda)+(\lambda+\varepsilon_2)^2(r_2-\lambda), \\ A_2 &= 2\Omega^2+(\lambda+\varepsilon_2)(-r_2+\lambda), \\ A_3 &= \Omega^2(-\varepsilon_1-2r_1+\lambda)+(\lambda+\varepsilon_1)(-r_1+\lambda), \\ A_4 &= -2\Omega^2+(-\lambda+r_1)(\lambda+\varepsilon_1), \\ B_1 &= \Omega^2(-\varepsilon_2+2r_2+3\lambda)+\varepsilon_2^2(r_2-\lambda)+\lambda(\lambda+\varepsilon_2)(r_2-2\lambda), \\ B_2 &= -2\Omega^2+(\lambda+\varepsilon_2)(-r_2+\lambda), \\ B_3 &= \Omega^2(\varepsilon_1-2r_1+3\lambda)+\varepsilon_1^2(-r_1+\lambda)+\lambda(\lambda+2\varepsilon_1)(-r_1+\lambda), \\ B_4 &= 2\Omega^2+(-\lambda+r_1)(\varepsilon_1+\lambda). \end{aligned}$$

Here S_0 is the power density at zero frequency, $S_1(\omega)$ is the power density connected with the noise background, $S_2(\omega)$ is the power density associated with the output signal and $(r_1 \leftrightarrow r_2)$ is obtained from the third term by interchanging r_1 and r_2. We give the expression of signal-to-noise ratio R as the ratio of the output power densities of the signal and the noise background at the frequency $\omega = \Omega$. Therefore, the signal-to-noise R is

$$R = \frac{\int_0^\infty S_2(\omega)\delta(\omega-\Omega)\,\mathrm{d}\omega}{S_1(\omega=\Omega)}, \tag{29}$$

where $S_i(\omega)$ $(i = 1, 2)$ were defined earlier.

It should be pointed out that equation (29) is obtained by using approximations (19) and (20), but its applicability is not restricted within small noise intensity and amplitude of signal.

3. Discussion and conclusion

According to expression (29) of SNR, we will discuss the influences of noise and signal on the signal-to-noise ratio and draw some conclusions.

3.1. Discussion

By virtue of the equation of the signal-to-noise ratio (29), the effects of the multiplicative noise intensity σ_1, frequency Ω of an external field, asymmetry Δ_1 of multiplicative noise and the correlation time $\tau = \lambda^{-1}$ of noise on the SNR are discussed through figures 1–5.

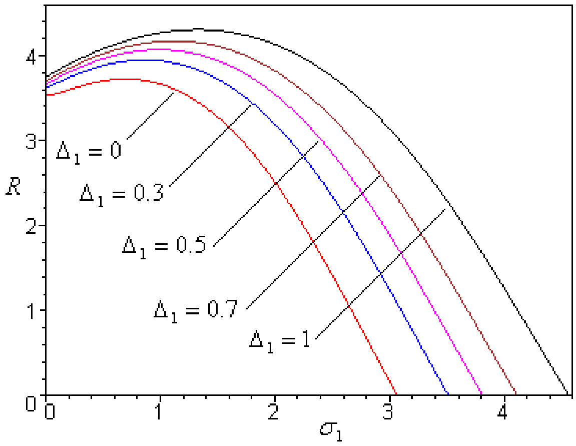

Figure 1. SNR as a function of the multiplicative noise intensity σ_1 for $\alpha = 1$, $a = 2$, $A = 1$, $\sigma_2 = 2$, $\sigma_3 = 1$, $\Delta_2 = 1$, $\Omega = 1$, $\lambda = 1$ with varied Δ_1.

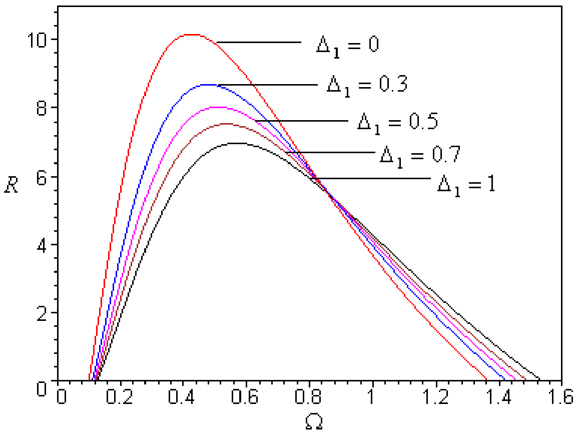

Figure 2. SNR as a function of Ω for $\alpha = 1$, $a = 2$, $A = 1$, $\sigma_1 = \sigma_3 = 1$, $\sigma_2 = 2$, $\Delta_2 = 1$, $\lambda = 1$ with varied Δ_1.

In figure 1, we plot the curve of SNR versus the multiplicative noise intensity σ_1 with varied asymmetry Δ_1 of multiplicative noise. The curve exhibits a maximum and SNR is a non-monotonic function of σ_1; the conventional stochastic resonance phenomenon occurs in this case, which is absent in [11]. At the same time, the SNR increases with the increase of Δ_1.

The curve of the SNR versus the frequency Ω with varied asymmetry Δ_1 of multiplicative noise is plotted in figure 2. It is seen that the curves exhibit a pronounced single peak, and the bona fide stochastic resonance exists. The SNR decreases with the increase of Δ_1 when $\Omega < 0.84$, while the SNR increases with the increase of Δ_1 when $\Omega \geqslant 0.84$. Therefore, the asymmetry Δ_1 of multiplicative noise reduces the SNR for small frequency ($\Omega < 0.84$), but enhances the SNR for large frequency ($\Omega \geqslant 0.84$). The SNR increases with the increase of Δ_1 in figure 1 for $\Omega = 1 (> 0.84)$, so figure 1 is consistent with figure 2.

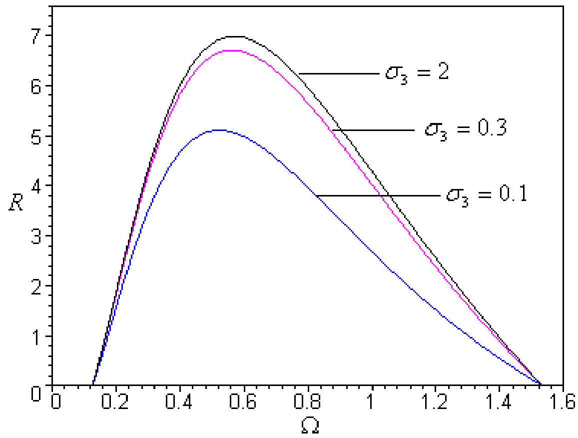

Figure 3. SNR as a function of frequency Ω for $\alpha = 1$, $a = 2$, $A = 1$, $\sigma_2 = 2$, $\sigma_1 = 1$, $\Delta_1 = 1$, $\Delta_2 = 1$, $\lambda = 1$ with varied σ_3.

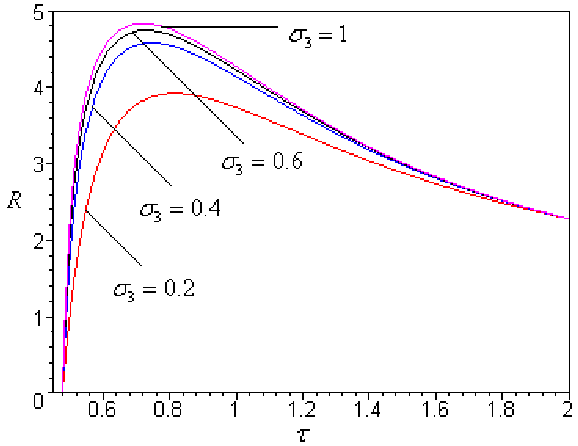

Figure 4. SNR as a function of τ for $\alpha = 1$, $\Omega = 1$, $a = 2$, $A = 1$, $\sigma_2 = 2$, $\sigma_1 = 1$, $\Delta_1 = 1$, $\Delta_2 = 1$ with varied σ_3.

Figure 3 is a plot of the SNR as a function of the frequency Ω with different cross-correlation strength σ_3 between the noises $\xi(t)$ and $\eta(t)$. We show that the SNR is a non-monotonic function of Ω. The SNR increases with the increase of σ_3. However, the increment of the SNR becomes smaller and smaller when σ_3 becomes larger. So the SNR increases monotonically with increasing σ_3 at first, but finally becomes saturated.

In figure 4, we plot the curves of the SNR versus the correlation time τ of noise with varied σ_3. Each of the curves exhibits a maximum and non-monotonic behaviour, so the stochastic resonance occurs in a broad sense. The SNR increases with the increase of σ_3. From figures 3 and 4, the cross-correlation strength σ_3 between the noises $\xi(t)$ and $\eta(t)$ can enhance the SNR and improve the output signal.

The curve of the SNR versus the correlation time τ of noise with varied σ_2 is given in figure 5. When we fix the intensity of multiplicative noise and the cross correlation strength between the two noises, the SNR decreases with the increase of additive noise intensity σ_2.

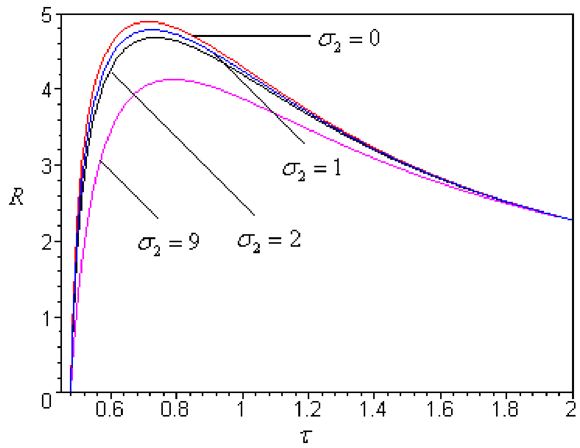

Figure 5. SNR as a function of τ for $\alpha = 1$, $a = 2$, $A = 1$, $\sigma_1 = 1$, $\sigma_3 = 0.5$, $\Delta_1 = 1$, $\Delta_2 = 1$, $\Omega = 1$ and with varied σ_2.

That is to say that the additive noise weakens the SNR and has a destructive effect on improving the output signal.

3.2. Conclusions

We have studied SR in an over-damped linear system with multiplicative noise, signal-modulated noise and additive noise. Appling the theory of SNR and Shapiro–Loginov formula, the expression of SNR is derived for asymmetric dichotomous noise. It is found that three different forms of stochastic resonance exhibit in this linear system: conventional SR, bona fide SR and SR in the broad sense. Moreover, the SNR decreases with the increase of Δ_1 when $\Omega < 0.84$ while the SNR increases with the increase of Δ_1 when $\Omega \geqslant 0.84$. The SNR increases with increasing σ_3, but decreases with increasing σ_2.

It is important to establish the existence of the conventional SR, not just SR in the broad sense, in such a linear system. Moreover, the noise $\eta(t)$ in equation (1) is divided into two parts: one is the additive noise, and the other is modulated by periodic signal. So the processes are different for direct signal modulation and bias signal modulation. We have studied the former in another paper.

The modulated noise can be used in many fields, such as optical communication and radio astronomy, to describe the fluctuation. Therefore the results in this paper provide a theoretical basis to the study of optical amplifier and optical communication systems.

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References

[1] Benzi R, Sutera A and Vulpiani A 1981 *J. Phys. A: Math. Gen.* **14** L453
[2] Nicolis C and Nicolis G 1981 *Tellus* **33** 225
[3] Fauve S and Heslot F 1983 *Phys. Lett.* A **97** 5

[4] McNamara B, Wiesenfeld K and Roy R 1988 *Phys. Rev. Lett.* **60** 2626
[5] McNamara B and Wiesenfeld K 1989 *Phys. Rev.* A **39** 4854
[6] Dykman M I, Mannella R, McClintock P V E and Stocks N G 1990 *Phys. Rev. Lett.* **65** 2606
[7] Hu G, Nicolis G and Nicolis C 1990 *Phys. Rev.* A **42** 2030
[8] Zhou T, Moss F and Jung P 1990 *Phys. Rev.* A **42** 3161
[9] Fuliński A 1995 *Phys. Rev.* E **52** 4523
[10] Berdichevsky V and Gitterman M 1996 *Europhys. Lett.* **36** 161
[11] Berdichevsky V and Gitterman M 1999 *Phys. Rev.* E **60** 1494
[12] Gitterman M 2003 *Phys. Rev.* E **67** 057103
[13] Gitterman M 2004 *Phys. Rev.* E **69** 041104
[14] Dykman M I, Luchinsky D G, McClintock P V E and Stein N D 1992 *Phys. Rev.* A **46** R1713
[15] Wang J, Cao L and Wu D J 2004 *Chin. Phys. Lett.* **21** 246
[16] Fuliński A and Góra P F 1993 *Phys. Rev.* E **48** 3510
[17] Shapiro V E and Loginov V M 1978 *Physica* A **91** 563
[18] Fuliński A 1995 *Phys. Rev.* E **52** 4523
[19] Fuliński A 1994 *Phys. Rev.* E **50** 2668
[20] Fuliński A and Góra P F 2001 *Phys. Rev.* E **64** 011905